

The phase diagram in the SU(3) Nambu-Jona-Lasinio model with 't Hooft and eight-quark interactions.

J. Moreira*, B. Hiller*, A. A. Osipov^{*,†} and A. H. Blin*

*Centro de Física Computacional, Departamento de Física da Universidade de Coimbra, 3004-516 Coimbra, Portugal

[†]Dzheleпов Laboratory of Nuclear Problems, JINR 141980 Dubna, Russia

Abstract. It is shown that the endpoint of the first order transition line which merges into a crossover regime in the phase diagram of the Nambu–Jona-Lasinio model, extended to include the six-quark 't Hooft and eight-quark interaction Lagrangians, is pushed towards vanishing chemical potential and higher temperatures with increasing strength of the OZI-violating eight-quark interactions. We clarify a connection between the location of the endpoint in the phase diagram and the mechanism of chiral symmetry breaking at the quark level. Constraints on the coupling strengths based on groundstate stability and physical considerations are explained.

Keywords: Spontaneous chiral symmetry breaking, general spin 0 eight-quark interactions, QCD phase diagram, finite temperature and chemical potential

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INTRODUCTION

The determination of the phase diagram of QCD represents an interesting challenge both from the experimental and the theoretical points of view. On the experimental side several new facilities are aiming at an unprecedented look at the properties of hot and/or dense strongly interacting matter and from the theoretical point of view, as standard perturbative techniques fail in the low energy regime of QCD, alternative approaches have to be used. Great progress has been achieved over the last two decades in the theoretical study of the QCD phase diagram both in terms of effective low energy theories and lattice calculations (see for instance reviews [1, 2, 3, 4] and paper [5]).

The Nambu–Jona-Lasinio model [6, 7, 8], is regarded as an useful tool for the study of low energy hadron phenomenology as it shares with QCD the global symmetries and incorporates by construction a mechanism for dynamical chiral symmetry breaking (DχSB). The extension to include a $2N_f$ 't Hooft determinantal interaction (NJLH) [9, 10, 11, 12, 13] (where N_f is the number of flavors) breaks the unwanted $U_A(1)$ symmetry exhibited by this model. The study of this model in the light quark sector (u, d and s) has shown a fundamental flaw [14] (the absence of a globally stable ground state) which can be remedied with the inclusion of general scalar non derivative $U_L(3) \times U_R(3)$ symmetric eight quark interactions.

FORMALISM

The model Lagrangian that we are going to use, \mathcal{L}_{eff} , can be decomposed in the following manner (see (1)):

$$\begin{aligned}\mathcal{L}_{eff} &= \mathcal{L}_D + \mathcal{L}_{NJL} + \mathcal{L}_H + \mathcal{L}_{8q} \\ \mathcal{L}_D &= \bar{q} (i\gamma^\mu \partial_\mu - m) q & \mathcal{L}_{8q} &= \mathcal{L}_{8q}^{(1)} + \mathcal{L}_{8q}^{(2)} \\ \mathcal{L}_{NJL} &= \frac{G}{2} \left[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5 \lambda_a q)^2 \right] & \mathcal{L}_{8q}^{(1)} &= 8g_1 [(\bar{q}_i P_R q_m)(\bar{q}_m P_L q_i)]^2 \\ \mathcal{L}_H &= \kappa (\det \bar{q} P_L q + \det \bar{q} P_R q) & \mathcal{L}_{8q}^{(2)} &= 16g_2 (\bar{q}_i P_R q_m)(\bar{q}_m P_L q_j)(\bar{q}_j P_R q_k)(\bar{q}_k P_L q_i). \end{aligned} \quad (1)$$

The free Dirac term, \mathcal{L}_D , contains a current mass matrix, m , diagonal in flavor space which explicitly breaks chiral symmetry (we will consider $m_u = m_d \neq m_s$ as such $SU(3)$ flavor symmetry is also broken). The 4-quark NJL interaction, \mathcal{L}_{NJL} , if strong enough induces spontaneous chiral symmetry breaking (here $\lambda_0 = \sqrt{2/3}$ and the λ_a ,

$a = 1, \dots, 8$ are Gell-Mann matrices acting in flavor space). The 6-quark 't Hooft term, \mathcal{L}_H , breaks axial symmetry (here the $P_{L,R}$ are chiral projectors and the determinant is over flavor space). The 8-quark interaction, \mathcal{L}_{8q} , can be split in two parts, one of which, $\mathcal{L}_{8q}^{(1)}$, along with the 't Hooft term, induces OZI rule violation.

This additional term ensures ground state stability as long as: $g_1 > 0$, $g_1 + 3g_2 > 0$ and $g_1 > 1/G(\kappa/16)^2$ [14]. Through a suitable choice of parameters (a decrease in G with increased g_1 with remaining parameters kept fixed) the light scalar and pseudoscalar meson spectra in the vacuum can be left relatively unchanged, apart from a marked decrease in the σ meson mass, and close to the one obtained in the NJLH model [15].¹

This wide spectrum of allowed $g_1(G)$ encompasses different scenarios ranging from the low g_1 (high G) case where, similarly to what happens in the NJLH model, the NJL term is the responsible for $D\chi$ SB, to the high g_1 (low G) scenario the NJL term is too weak to induce it and the physical vacuum is in fact induced by the 6-quark interactions.

These different scenarios can be disentangled by considering the non-vacuum properties of the model. This can be seen in study of the effects of the inclusion of a constant magnetic field [16, 17] or of finite temperature [18, 19] and/or chemical potential which are to a large extent dictated by the relative strength of the 4-quark and 8-quark interactions. Our focus will be on the latter since we are concerned with the phase diagram of QCD.

For $N_f > 2$ the bosonization of this effective quark Lagrangian can be done in the path integral formalism with the aid of the introduction of two sets of flavour nonet bosonic fields: $\{\sigma_a, \phi_a\}$, related to the physical scalar and pseudo-scalar mesons, and $\{s_a, p_a\}$ which are auxiliary fields [13]. The vacuum-to-vacuum amplitude can then be split in two parts, enabling the separate evaluation of the functional integral over the fermionic degrees of freedom and over the auxiliary variables. The former can be done using a symmetry preserving heat kernel scheme (generalized to deal with non-degenerate quark current masses [20, 21, 22]) whereas the latter can be done using the stationary phase approximation (for the present results it was done to leading order).

The minimization of the effective (or thermodynamic) potential then corresponds to the self consistent solution of two sets of equations (for details see for instance [15] and references therein). One of these comes from demanding vanishing vacuum expectation value for the meson fields, the gap equations,

$$\begin{cases} \Delta_u + Gh_u + \frac{\kappa}{16}h_d h_s + \frac{g_1}{4}h_u(h_u^2 + h_d^2 + h_s^2) + \frac{g_2}{2}h_u^3 = 0 \\ \Delta_d + Gh_d + \frac{\kappa}{16}h_u h_s + \frac{g_1}{4}h_d(h_u^2 + h_d^2 + h_s^2) + \frac{g_2}{2}h_d^3 = 0 \\ \Delta_s + Gh_s + \frac{\kappa}{16}h_u h_d + \frac{g_1}{4}h_s(h_u^2 + h_d^2 + h_s^2) + \frac{g_2}{2}h_s^3 = 0 \end{cases}, \quad (2)$$

while the other comes from the Stationary Phase requirements to integrate out auxiliary fields $\{s_a, p_a\}$,

$$\begin{cases} \frac{1}{2}h_u + \frac{N_c M_u}{24\pi^2} (6I_0 - (\Delta_{us} + \Delta_{ds} + 3\Delta_{ud})I_1) = 0 \\ \frac{1}{2}h_d + \frac{N_c M_d}{24\pi^2} (6I_0 - (\Delta_{us} + \Delta_{ds} - 3\Delta_{ud})I_1) = 0 \\ \frac{1}{2}h_s + \frac{N_c M_s}{24\pi^2} (6I_0 + 2(\Delta_{us} + \Delta_{ds})I_1) = 0 \end{cases}. \quad (3)$$

Here $\Delta_i = M_i - m_i$, with M_i , the dynamical mass of the quark species i and the Δ_{ij} are differences of squared masses $\Delta_{ij} = M_i^2 - M_j^2$. The I_i correspond to the averaged sum of $I_i = 1/3 \sum_{i=u,d,s} J_i$, where the J_i are quark 1-loop euclidean momentum integrals with $i+1$ vertices. For their regularization we chose to use two Pauli-Villars subtractions resulting in the definition:

$$J_i = 16\pi^2 \Gamma(i+1) \int \frac{d^4 p_E}{(2\pi)^4} \hat{\rho}_\Lambda \frac{1}{(p_E^2 + M^2)^{i+1}}$$

$$\hat{\rho}_\Lambda = 1 - (1 - \Lambda^2 \frac{\partial}{\partial M^2}) e^{\Lambda^2 \frac{\partial}{\partial M^2}} \quad (4)$$

Integrals of different orders can be easily related through recursion relations: $J_{i+1} = -\partial_{M^2} J_i$. Their generalization to take into account the effect of finite temperature and/or chemical potential is done through the introduction of the Matsubara frequencies as per usual.

¹ We chose to fit the model parameters G , κ , g_2 , Λ , $m_u(=m_d)$, m_s to reproduce the following vacuum properties: $M_\pi = 0.138\text{GeV}$, $M_K = 0.494\text{GeV}$, $M_{\eta'} = 0.958\text{GeV}$, $M_{a_0} = 0.980\text{GeV}$, $f_\pi = 0.092\text{GeV}$ and $f_K = 0.117\text{GeV}$. This sets the stability bounds on the value for the coupling strength $347\text{GeV}^{-8} = g_1^{(1)} < g_1 < g_1^{(2)} = 20300\text{GeV}^{-8}$. One might naively expect another constraint coming from the requirement that M_σ is a real positive but the critical value for that to be false lies outside the stability conditions: $g_1^{(0)} = 20933\text{GeV}^{-8}$.

The thermodynamic (or effective) potential can be obtained by integration of the gap equations, resulting in (see [23] for details):

$$\Omega = \frac{1}{32} (8G(h_u^2 + h_d^2 + h_s^2) + 2\kappa h_u h_d h_s + 3g_1(h_u^2 + h_d^2 + h_s^2) + 6(h_u^4 + h_d^4 + h_s^4)) + \frac{3N_c}{8\pi^2} (J_{-1}(M_u, \Lambda, \mu_u, T) + J_{-1}(M_d, \Lambda, \mu_d, T) + J_{-1}(M_s, \Lambda, \mu_s, T)). \quad (5)$$

The quark loop integral contribution, the J_{-1} integrals are given by:

$$J_{-1}(M, \Lambda, T, \mu) = -8 \int_0^\infty d|\vec{p}_E| (|\vec{p}_E|^2 \hat{p}_\Lambda E_p) - \frac{8}{3} \int_0^\infty d|\vec{p}_E| \left(|\vec{p}_E|^4 \hat{p}_\Lambda \frac{n_q + n_{\bar{q}}}{E_p} \right) - [\dots]_{M=0} \\ = J_{-1}(M, \Lambda) - \frac{8}{3} \int_0^\infty d|\vec{p}_E| \left(|\vec{p}_E|^4 \hat{p}_\Lambda \frac{n_q + n_{\bar{q}}}{E_p} \right), \quad (6)$$

where the last subtraction corresponds to the integrals evaluated with $M = 0$. In the last line we have made explicit the separation between the vacuum and the medium contribution. The latter involves (anti-)quark occupation numbers which are defined as:

$$n_q = \frac{1}{1 + e^{\frac{E_p - \mu}{T}}}, \quad n_{\bar{q}} = \frac{1}{1 + e^{\frac{E_p + \mu}{T}}}. \quad (7)$$

RESULTS

As was mentioned above there is a wide range of allowed values for g_1 (and G , as they have to be changed in consonance for the meson spectra to be left more or less unchanged) if we take into account only vacuum stability restrictions. In [18, 19] it was shown that the temperature behaviour at vanishing chemical potential changes dramatically over this range. The OZI violating eight quark interactions dictate the temperature, slope and nature of the chiral transition: higher g_1 results in lower T_c , with a steeper slope and above a certain critical value one obtains a first order transition (which is made possible by the above described scenario of a physical vacuum induced by the 't Hooft term in a potential that would otherwise correspond to a distorted Wigner-Weyl phase). As there is growing evidence coming from lattice calculations [24, 25] that the transition at $\mu = 0$ is crossover this can be interpreted as another upper bound for g_1 ².

In the case for zero temperature and finite chemical potential (considered equal for all quark species $\mu = \mu_u = \mu_d = \mu_s$), on the other hand we always find a first order chiral transition for some critical value μ_c which is lower for higher g_1 . The analysis of the dependence of the quarks dynamical mass as a function of the chemical potential for different values of g_1 (see Fig. 1) reveals that, below a certain critical value $g_1^{(4)}$, the model describes a low density phase of massive quarks (notice that on the left panel the line referring to $M_{u,d}(\mu)$ crosses the diagonal before the first order transition jump). As this is deemed unphysical (see for instance [26]) this sets a lower bound for the interaction strength³.

Plotting the dynamical mass as a function on the bayonic density (as can be seen in the left panel of Fig. 2) we see furthermore that the jump in M_s is higher and occurs at lower densities for stronger coupling. Furthermore the critical chemical potential for non-vanishing strange quark density is lower for higher g_1 . Both these results may have interesting implications in the physics of compact stars.

For finite T and μ (see Fig. 3) we find that the crossover/first order chiral transition line and the position of the Critical End Point (CEP) are highly sensitive to the chosen value for g_1 . As the transition as a function of T is steeper for higher g_1 the critical μ for it to become first order is lower for higher g_1 : the CEP moves to the left in the phase diagram with increasing OZI-violating eight quark interactions. Furthermore in the studied cases the transition line, $T_c(\mu)$, for higher g_1 lies completely inside that of lower g_1 .

² Using the already mentioned fitting procedure we obtain $g_1^{(3)} = 8703 \text{ GeV}^{-8}$ which is lower than the upper bound required by vacuum stability.

³ For the chosen fits we have $g_1^{(4)} = 2423 \text{ GeV}^{-8}$ which is also inside the range allowed by vacuum stability

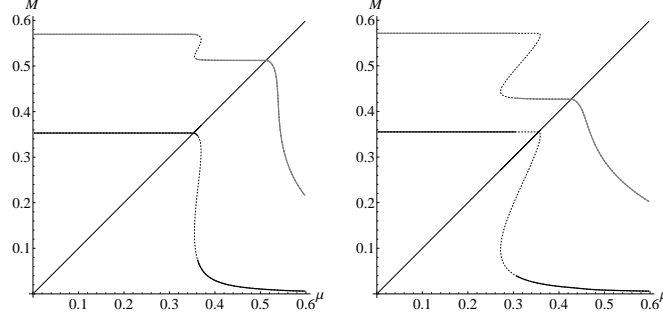


FIGURE 1. Dynamical mass of the quarks as a function of chemical potential at zero temperature ($[M] = [\mu] = \text{GeV}$). Left panel refers to the weak coupling case set a) of Table 1 whereas the right panel refers to set b). The upper grey thick lines refer to M_s , the black thick lines refer to M_u and the dotted line refers to unstable solutions which are skipped in the process of the first order transition. The diagonal line refers to $M = \mu$.

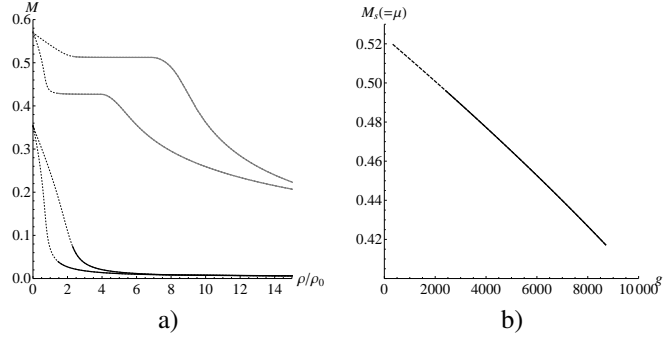


FIGURE 2. **a)** Dynamical mass of the quarks ($[M] = \text{GeV}$) as a function of baryonic density (divided by $\rho_0 = 0.17\text{fm}^{-3}$) at zero temperature. The upper grey and black lines correspond respectively to M_s and $M_{u,d}$ in the weak coupling parameter set (the lower correspond to stronger coupling). The dotted lines correspond to unstable solutions which are skipped by the first order transition. **b)** Chemical potential above which the density of strange quarks becomes non-vanishing as a function of g_1 . The dashed line corresponds to the range of g_1 values allowed by vacuum stability but excluded by the requirement of the unexistence of a low-density phase of massive quarks. The full line goes through the allowed values up to the limit that corresponds to the requirement of crossover transition at zero chemical potential.

CONCLUSIONS

The eight quark interactions necessary to stabilize the ground state in three flavor NJLH model, dictate the phase diagram of the model. Their strength can be constrained using considerations on vacuum stability and the requirements of crossover at vanishing chemical potential as well as the unexistence of a low density phase of massive quarks.

TABLE 1. Parameter sets obtained by fits to the π , κ , η' and a_0 masses, as well as the π and K weak decay constants. The value for OZI violating part of the eight quark interactions (marked with *) is not fitted.

Sets		m_u (MeV)	m_s (MeV)	M_u (MeV)	M_s (MeV)	Λ (MeV)	G (GeV^{-2})	κ (GeV^{-5})	g_1 (GeV^{-8})	g_2 (GeV^{-8})
a		5.9	186	359	554	851	10.92	-1001	1000*	-47
b		5.9	186	359	554	851	7.03	-1001	8000*	-47

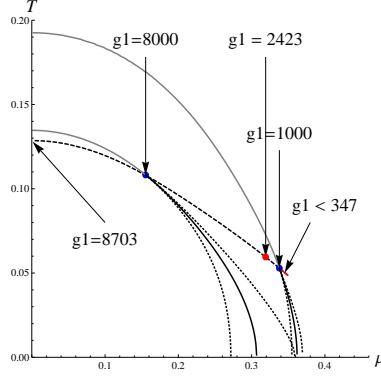


FIGURE 3. Phase diagram for different values of g_1 , given in units of GeV^{-8} . Temperature, T , and chemical potential, μ , are given in GeV . The gray lines correspond to the crossover transition for the two parameter sets from Table 1 (outermost corresponds to weaker coupling). The blue dots, the full and dotted black lines correspond (again for the two sample parameter sets) respectively to: CEP, first order transition and spinodals. The red dot marks the position of the CEP for the critical value below which a low density gas of massive quarks is obtained $T = 0$, $g_1 = 2423 \text{ GeV}^{-8}$. The dashed line corresponds to the range of allowed CEP.

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